THE ULTIMATE BEARING CAPACITY OF FOUNDATIONS

by

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SYNOPSIS

In the first part of the article a theory of bearing capacity is developed, on the basis of plastic theory, by extending the previous analysis for surface footings to shallow and deep foundations in a uniform cohesive material with internal friction. The theoretical results are represented by bearing capacity factors in terms of the mechanical properties of the soil, and the physical characteristics of the foundation. The base resistance of foundations in purely cohesive material is found to increase only slightly with foundation depth; for deep foundations the skin friction is, therefore, large compared with the base resistance. In cohesionless material, however, the base resistance increases rapidly with foundation depth and depends to a considerable extent on the earth pressure coefficient on the shaft; for deep foundations the base resistance is the predominant feature and the skin friction is relatively small.

In the second part of the article the main results of laboratory and field loading tests on buried and driven foundations are analysed and compared with the theoretical estimates. The observed base resistance of foundations in clay is in good agreement with the estimates; for deep foundations in soft clay the actual base resistance is somewhat less than estimated, on account of local shear failure, and an empirical compressibility factor is introduced by which the shearing strength is reduced. The skin friction is found to depend largely on the method of installing the foundation. The observed bearing capacity of shallow foundations in sand is in reasonable agreement with the theory; for deep foundations, however, the actual base resistance is considerably less than estimated on account of local shear failure, and an empirical compressibility factor is introduced as before. Since the earth pressure coefficient on the shaft can at present only be deduced from the skin friction of penetrating tests, it is frequently more convenient to estimate the bearing capacity of deep foundations in cohesionless soil from an extrapolation of the results of cone penetration tests.

Dans la première partie de l'article on expose une théorie sur la capacité de portage, basée sur la théorie de la plasticité, par extension de l'analyse préalable des empattements de surface aux fondations faibles et profondes dans une matière cohésive uniforme avec friction interne. Les résultats théoriques sont représentés par les facteurs de capacité de portage en fonction des propriétés mécaniques du sol et des caractéristiques physiques de la fondation. La résistance de base des fondations dans un sol vraiment cohésif ne s'accroît que faiblement avec la profondeur des fondations; pour les fondations profondes le frottement superficiel est donc grand par comparaison avec la résistance de base. Cependant, dans des matières sans cohésion, la résistance de base s'accroît rapidement avec la profondeur de fondation et dépend pour une grande mesure du coefficient de pression de la terre sur la souche; pour les fondations profondes la résistance de base est un facteur de première importance et le frottement superficiel n'a que peu d'importance.

Dans la deuxième partie de l'article, on peut voir l'analyse des principaux résultats d'essais de charge en laboratoire et sur le terrain, sur fondations enterrées et enfoncées, et la comparaison avec les prévisions théoriques. La résistance de base observée des fondations dans l'argile concorde bien avec les évaluations; pour les fondations profondes dans l'argile molle, la résistance de base réelle est quelque peu moindre que celle estimée en raison du manque de résistance locale au-cisaillement, et on introduit un facteur empirique de compressibilité par lequel la résistance au cisaillement est réduite. On trouve que le frottement superficiel dépend beaucoup sur la méthode d'installation des fondations. La capacité de portage observée pour les fondations peu profondes dans le sable concorde raisonnablement avec la théorie; pour les fondations profondes, cependant, la résistance de base réelle est bien moindre que celle estimée en raison du manque de résistance locale au cisaillement et un facteur empirique de compressibilité est introduit comme ci-dessus. Comme le coefficient de pression de la terre sur la souche ne peut à l'heure actuelle être déduit que d'après le frottement superficiel des essais de pénétration, il est souvent plus commode d'estimer la capacité de portage des fondations profondes en terrain sans cohésion d'après une extrapolation des résultats des essais de pénétration au cône.

INTRODUCTION

The design of foundations must satisfy two main requirements, namely, complete failure of the foundation must be avoided with an adequate margin of safety, and the total and relative settlements of the foundation must be kept within limits that can be tolerated by the superstructure. This article is only concerned with complete failure of the foundation, or its ultimate bearing capacity, on which a factor of safety of 3 is generally used in practice to determine the maximum safe foundation load. The settlement of the foundation under

this load has to be estimated independently to ascertain its effect on the stresses in the superstructure.

The ultimate bearing capacity of a foundation is defined as the maximum load that the ground can sustain (general shear failure) ; where the load-settlement curve does not exhibit a peak load, the bearing capacity is taken as the load at which the curve passes into a steep and fairly straight tangent (local shear failure) (Terzaghi, 1943). A theoretical method for estimating this bearing capacity is outline in the first part of the article for cohesive materials with internal friction, and for special cases of purely cohesive and cohesionless materials. The main results of laboratory and field loading tests in clay and sand are summarized, and compared with the theoretical estimates, in the second part of the article.

PART I. THEORY OF BEARING CAPACITY

BEARING CAPACITY OF COHESIVE MATERIAL WITH INTERNAL FRICTION

The bearing capacity of foundations depends on the mechanical properties of the soil (density, shearing strength and deformation characteristics), on the original stresses and the water conditions in the ground, on the physical characteristics of the foundation (size, depth, shape and roughness) and on the way in which the foundation is installed. In view of mathematical difficulties the problem can at present only be solved by simplified methods. This article only considers single vertical loads acting centrally on foundations with a horizontal base resting in a homogeneous material of great depth.

The bearing capacity of surface and shallow foundations is generally estimated on the assumption that the soil is a rigid material (general shear failure) ; for deep foundations, when the deformation characteristics become of greater importance, the compressibility of the material is usually taken into account by an empirical reduction of the shearing strength (local shear failure). The influence on the soil properties of the method of installing the foundation is also based on empirical evidence.

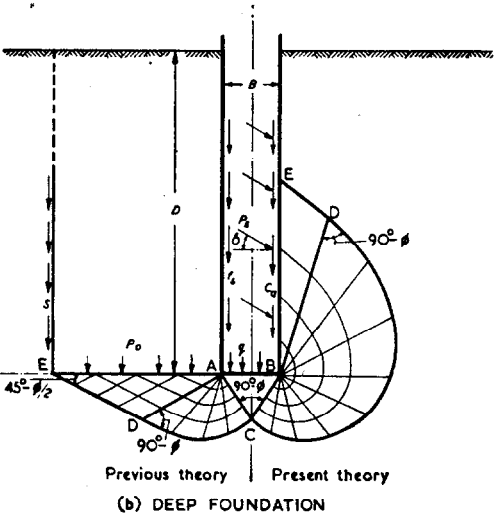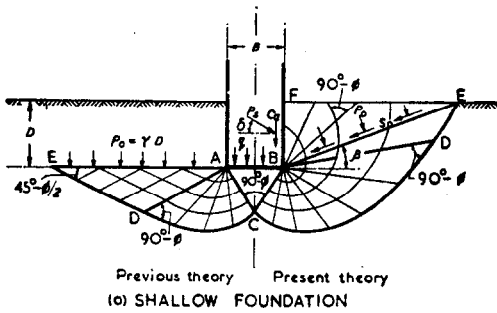For a material whose shearing strength is given by the equation

$$s = c + p \tan \phi \text{ (Coulomb-Mohr's}$$
$$\text{theory of rupture) . . (1)}$$

where c denotes unit or apparent cohesion,
         ϕ ,,   angle of internal friction or shearing resistance,
and p ,,   normal pressure on shear plane,

Terzaghi (1943) has shown that the bearing capacity q of a shallow strip foundation of width B and depth D (Fig. 1 (a)) can be represented by the expression

$$q = cN_c + p_0N_q + \gamma\frac{B}{2}N_\gamma \quad . \quad (2)$$

Figs 1

Previous theory   Present theory
(a) SHALLOW FOUNDATION

Previous theory   Present theory
(b) DEEP FOUNDATION

Plastic zones near rough strip foundation

where p_0 denotes overburden pressure at base level,

γ ,, density of material,

and N_c, N_q and N_γ are the bearing capacity factors for a surface footing and depend on ϕ and the roughness of the base.

For a similar deep foundation (Fig.1 (b)) Terzaghi has indicated that the bearing capacity is approximately equal to that given above, with the additional effects of the skin friction along the foundation shaft and the shearing stress along a vertical outer boundary of the mass of soil adjacent to the foundation.

The above analysis is based on plastic theory, and the corresponding zones of plastic equilibrium in the material are shown in Figs 1 for the case of a rough foundation. Below the base is a central zone ABC, which remains in an elastic state of equilibrium and acts as part of the foundation ; on each side of this zone there are two plastic zones, i.e. a zone of radial shear, ACD, and a zone of plane shear, ADE, identical to those below a similar surface footing. In the case of a shallow foundation the shearing strength of the overburden is ignored and only its weight is taken into account as an equivalent surcharge, p_0, equal to γD. This method has been found to be conservative, and the assumed mechanism of failure usually not in accordance with the observed ground movements (Meyerhof, 1948). For a deep foundation the corresponding method suffers from the difficulty that when the failure surface no longer reaches the ground level, the height over which the shearing strength of the soil is mobilized becomes very uncertain and must be assumed.

In an attempt to overcome these limitations the Author has extended the previous analysis of the plastic equilibrium of a surface footing to shallow and deep foundations. According to this theory the zones of plastic equilibrium increase with foundation depth to a maximum for a deep foundation (Figs 1). For a given depth the size of these zones varies with the roughness of the foundation, and for a perfectly smooth foundation two symmetrical plane shear zones are formed below the base. The extent of the zones is also largely governed by the shape of the foundation and is a minimum for a circular foundation (see p. 310)*.

At the ultimate bearing capacity the region above the composite failure surface is, in general, assumed to be divided into two main zones on each side of the central zone ABC (Fig. 1 (a)), namely a radial shear zone BCD and a mixed shear zone BDEF in which the shear varies between the limits of radial and plane shear, depending largely on the depth and roughness of the foundation. The plastic equilibrium in these zones can be established from the boundary conditions starting at the foundation shaft. To simplify the analysis, the resultant of the forces on the foundation shaft BF and the weight of the adjacent soil wedge BEF are replaced by the equivalent stresses p_0 and s_0, normal and tangential, respectively, to the plane BE. This plane may then be considered as an " equivalent free surface " subjected to the " equivalent free surface stresses " p_0 and s_0. The inclination β of the " equivalent free surface " increases with foundation depth and together with the " equivalent free surface stresses " forms therefore a parameter of that depth.

On this basis the bearing capacity can approximately be represented by the equation

$$q = cN_c + p_0 N_q + \gamma \frac{B}{2} N_\gamma. \qquad \ldots \ldots \ldots \quad (2)$$

This expression is of the same form as that given by Terzaghi (see above), but N_c, N_q and N_γ . are now the general bearing capacity factors which depend on the depth and shape of the foundation as well as ϕ and the roughness of the base. Since the investigation of the influence of the weight of the material on the characteristics of the plastic equilibrium has not yet passed beyond the stage of formulating the differential equations, the problem can at present only be solved in two stages :—

The first stage is an analytical treatment based on an extension of the work of Prandtl

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* Page numbers are those on which begin the sections to which the Author refers.

(1920) and Reissner (1924) ; this assumes a weightless material and for a part of the bearing capacity gives the equation

$$q' = cN_c + p_0N_q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The second stage is a semi-graphical treatment based on an extension of the work of Ohde (1938) ; this takes the weight of the material into account and for an approximate part of the bearing capacity gives the equation

$$q'' = \gamma \frac{B}{2} N_\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For a foundation with a perfectly smooth base q' is the same as that before but

$$q'' = \gamma \frac{B}{4} N_\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

Trial computations have shown that in practice the base can always be taken as perfectly rough, but the shaft can vary between being perfectly rough and perfectly smooth.

In each case the bearing capacity factors are first derived in terms of the foundation depth parameters (β, p_0 and s_0) ; these parameters are later determined explicitly for foundations of various depths. It has been found convenient to express the resultant bearing capacity by the relation

$$q = cN_{cq} + \gamma \frac{B}{2} N_{\gamma q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where one term represents the influence of the cohesion and the other represents the influence of the weight of the material. The factors N_{cq} (depending on N_c and N_q) and $N_{\gamma q}$ (depending on N_γ and N_q) are the resultant bearing capacity factors. It should be noted that the above expressions give only the base resistance of a foundation ; to this base resistance must be added any skin friction along the shaft to obtain the total bearing capacity of the foundation.

General bearing capacity factors N_c and N_q for strip foundation

The above procedure may be illustrated by computing the bearing capacity of a strip foundation with a rough base of width B. As indicated above the bearing capacity factors are first derived in terms of the foundation depth parameters (β, p_0 and s_0). The zones of plastic equilibrium corresponding to the general case are given in Fig. 2 (a), where the equivalent free surface AE produced is inclined at β and subjected to the equivalent free surface stresses p_0 and s_0, normally and tangentially, respectively.

In the plane shear zone ADE, with angle η at A, the plastic equilibrium requires that along AD and DE the shearing strength s_1, under the normal pressure p_1 is fully mobilized and is equal to $c + p_1 \tan \phi$. Hence from Mohr's diagram

$$\cos (2\eta + \phi) = \frac{s_0 \cos \phi}{c + p_1 \tan \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$= \frac{(c + p_0 \tan \phi)\, m \cos \phi}{c + p_1 \tan \phi} \quad . \quad . \quad . \quad . \quad (6a)$$
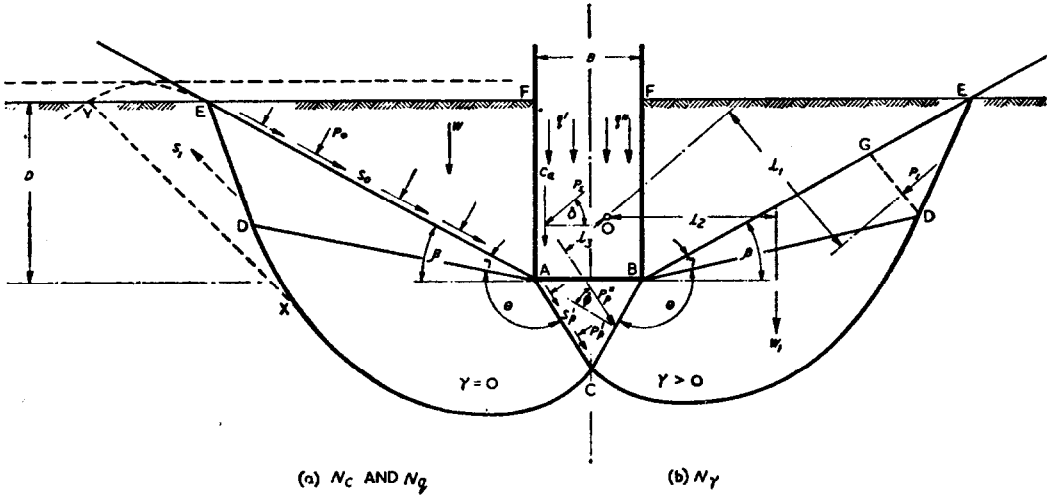
where m denotes degree of mobilization of shearing strength on equivalent free surface ($0 < m < 1$),

and $\qquad\qquad p_1 = \dfrac{c + p_1 \tan \phi}{\cos \phi} [\sin (2\eta + \phi) - \sin \phi] + p_0 \quad . \quad . \quad . \quad . \quad (7)$

from which η and p_1 can be determined for any given p_0, s_0, and ϕ.

In the radial shear zone ACD with angle $\theta = 135° + \beta - \eta - \phi/2$ at A, it can be shown

Figs 2

(a) N_C AND N_q (b) N_γ

Determination of general bearing capacity factors for strip foundation with rough base

(Prandtl, 1920) that CD is a logarithmic spiral surface, and that along this surface and radial sections the shearing strength is fully mobilized. In particular along AC the normal and tangential components of the passive earth are, respectively,

$$p_p' = (s_p' - c) \cot \phi \qquad \ldots \ldots \ldots \ldots \quad (8)$$

and

$$s_p' = (c + p_1 \tan \phi) \, e^{2\theta \tan \phi} \qquad \ldots \ldots \ldots \ldots \quad (9)$$

from which the bearing capacity is

$$q' = p_p' + s_p' \cot (45° - \phi/2) \qquad \ldots \ldots \ldots \quad (10)$$

Substituting equations (7) to (9) into (10)

$$q' = c \left[\cot \phi \left\{ \frac{(1 + \sin \phi) \, e^{2\theta \tan \phi}}{1 - \sin \phi \sin (2\eta + \phi)} - 1 \right\} \right] + p_0 \left[\frac{(1 + \sin \phi) \, e^{2\theta \tan \phi}}{1 - \sin \phi \sin (2\eta + \phi)} \right], \quad \ldots \quad (11)$$

or

$$q' = cN_c + p_0 N_q \qquad \ldots \ldots \quad \text{from equation (3)}$$

where N_c and N_q have the values given in the square brackets above.

To avoid determining the factors N_c and N_q in every case, they have been calculated for the lower limit of zero shearing stress on the equivalent free surface ($m = 0$) and for the upper limit of full mobilization of the shearing strength ($m = 1$), within practical limits of β and ϕ. The results (Figs 3 and 4) show that the factors increase rapidly with both β and ϕ and are not very sensitive to changes of m. It is of interest to note that for $m = 0$ the case $\beta = -90°$ represents confined (triaxial) compression, while for $-90° < \beta < 0$ the bearing capacity of a foundation on a slope is represented. The case $\beta = 0$ ($m = 0$) applies to a surface foundation, and the corresponding factors are identical to those derived for N_c by Prandtl (1920) and for N_q by Reissner (1924). For $0 < \beta < 90°$ the problem of shallow and deep foundations outlined above is represented. The limit of $\beta = 90°$ applies to a very deep foundation, and for the special case of $m = 0$ the factor N_c has also been obtained independently (Jaky, 1948).

Fig. 4
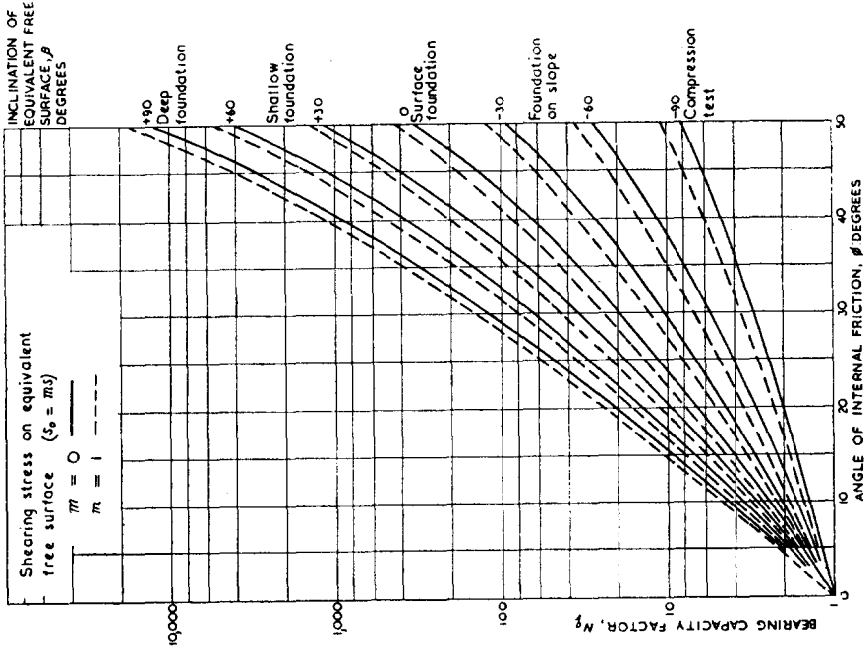
General bearing capacity factor N_q for strip foundation

Fig. 3
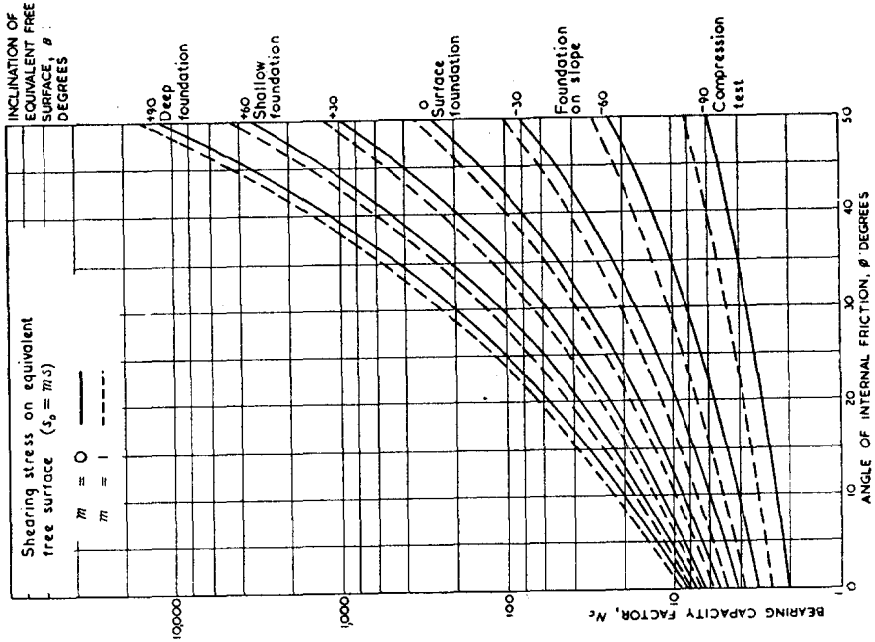
General bearing capacity factor N_c for strip foundation

General bearing capacity factor N_γ for strip foundation

A comparison of different general methods of estimating the bearing capacity has shown that the logarithmic spiral method (Ohde, 1938) is the most promising one, since it is rigorous if $\gamma = 0$ as indicated above and is reasonably close to the observed mechanism of failure. It has also been shown (Meyerhof, 1948) that in order to obtain the minimum factor, N_γ, it is essential at present to use a semi-graphical method for locating the worst centre of the spiral, since any restraint on the locus of this centre (e.g. Terzaghi, 1943) is open to objections. This method will therefore be used below ; it gives only an approximation to the actual plastic zones and the present failure surface is not necessarily continuous at the plane shear zones.

The procedure may be illustrated by determining the bearing capacity factor N_γ for a strip foundation as before. Considering forces to the right of the foundation centre line (Fig. 2 (b)) the plastic equilibrium is found by balancing the moments about any point **O** of the resistance P_1 due to the soil wedge **DEG**, which can be obtained from Mohr's diagram, the weight W_1 of the segment **BCDG** and the overturning resultant thrust P_p'' acting at an angle ϕ to the normal on the face **BC** and at $\frac{2}{3}$ **BC** from **B**. Thus

$$P_p'' = \frac{P_1 l_1 + W_1 l_2}{l_3} . \qquad \qquad \text{. . . . . (12)}$$

This analysis is repeated for different centres **O** of the spiral until the minimum value of P_p'' is found, which represents the total passive earth pressure. This procedure is rather laborious in practice since at least a dozen trials have to be made in any given case to determine the minimum resistance from which

$$q'' = \frac{\gamma B}{2} \left[\frac{4 \, P_p'' \sin \, (45° + \phi/2)}{\gamma B^2} - \tfrac{1}{2} \tan \, (45° + \phi/2) \right], \qquad \text{. . . . (13)}$$

or $\qquad\qquad q'' = \gamma \dfrac{B}{2} \, N_\gamma,$

from equation (4) where N_γ has the value given in the square brackets above.

The factor N_γ has been computed for the lower and upper limits ($m = 0$ and $m = 1$, respectively) within practical limits of β and ϕ as before. The results (Fig. 5) show that the factor increases rapidly with both β and ϕ, and is found to be practically independent of m. It may be noted that for $-\phi < \beta < 0$ the bearing capacity of a foundation on a slope is represented. The case $\beta = 0$ ($m = 0$) applies to a surface foundation and the factor is identical to that derived before (Meyerhof, 1948). For $\phi = 30°$ the present value of $N_\gamma = 22·9$ may be compared with a numerical step-by-step computation by Ohde (1938) ; in this he determined the passive earth pressure on an inclined rough wall tilted back at 30° with a slightly sloping ground surface ($\beta = -1·20°$). The Author has extended this computation to $\beta = 0$, and thus obtained from the vertical component of the passive pressure the factor $N_\gamma = 22·50$. The approximate estimate on the basis of the present theory agrees therefore within 2 per cent in this case. For $0 < \beta \leqslant 90°$ the problem of shallow and deep foundations is represented as before.

Resultant bearing capacity of strip foundation

In order to apply the above solutions to an estimate of the bearing capacity of a strip foundation at depth D, it is necessary to relate the foundation depth parameters (β, p_0 and s_0) to D. As a first approximation this relationship is determined on the assumption that the ground level passes through the intersection of the failure surface (corresponding to the above parameters) and the equivalent free surface (point **E**, Fig. 2 (a)). It can be shown that the corresponding foundation depth is given by the equation

$$D = \frac{\sin \beta \cos \phi \, e^{\theta \tan \phi} \, B}{2 \sin \, (45° - \phi/2) \cos \, (\eta + \phi)} \qquad \text{. . . . . . . (14)}$$

Fig. 5
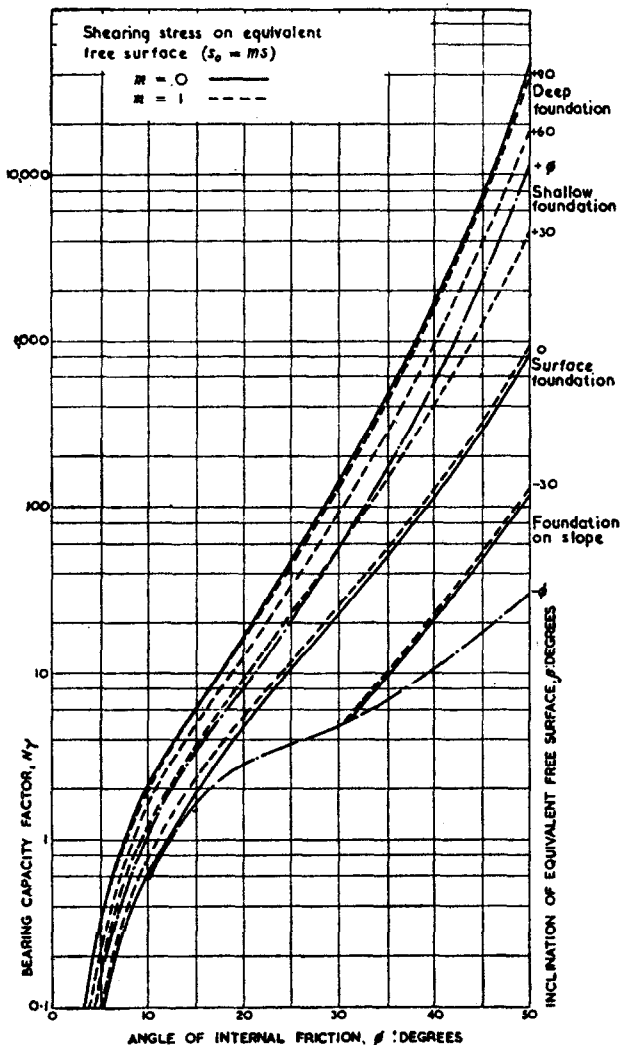
General bearing capacity factor N_γ for strip foundation

where β denotes inclination of equivalent free surface,

η and θ are the angles at A of the plane and radial shear zones, ADE and ACD, respectively (see p. 304).

The corresponding equivalent free surface stresses can be obtained from the forces on the foundation shaft and the weight of soil between the shaft and equivalent free surface. The forces on the shaft consist of two components, an adhesion C_a and a thrust P_s acting at the angle of skin friction δ (Fig. 2 (a)). Considering forces to the left of the foundation centre line,

$$C_s = c_a D \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (15)$$

and

$$P_s = p_s D$$

$$= K_s \, \gamma \, \frac{D^2}{2 \cos \delta} \quad \cdots \cdots \cdots \quad (16)$$

where c_a denotes unit adhesion,

      p_s ,,   average unit earth pressure on shaft within failure zone,

and K_s is the coefficient of earth pressure on shaft within failure zone. This coefficient depends on the mechanical properties of the material and the physical characteristics of the foundation (p. 312).

The weight of the soil wedge **AEF** between the shaft and equivalent free surface is given by the equation

$$W = \gamma \, \frac{D^2}{2} \cot \beta \quad \cdots \cdots \cdots \quad (17)$$

The normal and tangential components, P_0 and S_0, respectively, of the resultant force on the equivalent free surface can now be determined from the forces C_a, P_s and W whose magnitudes and directions are known, and thus give, for the average equivalent free surface stresses,

$$p_0 = \frac{P_0 \sin \beta}{D} \quad \cdots \cdots \cdots \quad (18)$$

and

$$s_0 = \frac{S_0 \sin \beta}{D} \quad \cdots \cdots \cdots \quad (19)$$

For a deep foundation ($\beta = 90°$) the stresses are :—

$$p_0 = p_s \cos \delta \quad \cdots \cdots \cdots \quad (18a)$$

and

$$s_0 = c_a + p_s \sin \delta \quad \cdots \cdots \cdots \quad (19a)$$

where the stresses are determined within the height of the failure surface (i.e. **BE** in Fig. 1 (*b*)) given by equation (14) for $\beta = 90°$.

When applying the above results to an estimate of the bearing capacity of a strip foundation (equation 2), it should be remembered that the factors N_c, N_q, and N_γ were derived for an equivalent free surface extending at a slope β beyond **E** (Figs 2). Since the ground surface is, however, horizontal beyond this point, the resistance outside the plastic zones may not be sufficient to withstand the stresses on the failure surface, particularly in the upper portion, and a greater foundation depth than that given by equation (14) may be required in order that the above bearing capacity factors hold. To ascertain whether a greater depth corresponds to these factors (i.e. to the particular slope β) the following approximate method is suggested :—

At any point **X** on the original failure surface (Fig. 2 (*a*)) the maximum acting force S_1 is equal to the total major principal stress between **E** and **X**, and in the direction **XY** of the resultant of this stress the resistance of the sliding block **EXY** of the material must be adequate. In this way a limiting curve, such as **EY**, can be constructed, and the vertical distance between the horizontal tangent to this curve and the foundation level gives the foundation depth required for adequate resistance of the material beyond the failure surface. If this depth is greater than that assumed (equation 14) the stresses p_0 and s_0 have to be increased accordingly (equations 15 to 19), and the analysis is repeated until the minimum foundation depth is found.

Trial investigations have shown that, for materials with an angle of internal friction of about 30° or more, the above method of adding algebraically the bearing capacity components (based on different failure surfaces) in accordance with equation (2), and using equation (14) for the foundation depth, gives results which are on the safe side when compared with a more rigorous method in which the bearing capacity components are added vectorially (based on a single failure surface) and the resistance beyond the failure surface is taken into account. In other cases (e.g. for a purely cohesive material) equation (14) is on the unsafe side, and the minimum foundation depth must be determined by a trial and error method such as the one suggested above.

Mechanism and extent of failure

Apart from an estimation of the bearing capacity of foundations, the theory gives also some indication of the probable extent of the failure surface. A solution of the problem of bearing capacity has been obtained in two stages. The first stage gives the part of the bearing capacity q' (factors N_c and N_q) by assuming a weightless material; the corresponding failure surface is obtained analytically and consists of plane and logarithmic spiral sections. The second stage gives the part of the bearing capacity q'' (factor N_y) due to the weight of the material (in the absence of cohesion and equivalent free surface stresses); the corresponding failure surface is obtained from a semi-graphical method and consists approximately of plane and logarithmic spiral sections although in reality it is a smooth continuous curve. This failure surface is much smaller than that above. In general, the failure surface corresponding to the resultant bearing capacity q probably lies, therefore, between the above limits.

Trial computations for a circular foundation on the surface and at great depth (p. 319) indicate that the failure surface is approximately circular; the extent of the failure is very much smaller than that of a similar strip foundation and is little affected by the angle of internal friction.

Since the theoretical movement of the material in the plastic zones is parallel to the failure surface, with increasing depth of the foundation the soil movement is thus changing from a general downward and outward direction to an upward one, which for a deep foundation is practically vertical; a movement towards the shaft is, however, unlikely in practice. Near a smooth shaft the particles are moving upwards while along a rough foundation the particles are dragged down with a shear plane on the interface.

Effect of deformation characteristics of material on failure condition

While in the above theory the material is assumed to be rigid (general shear failure), the soil in the plastic zones, and for some distance beyond the failure surface, is actually compressed and may be subject to volume changes, which accommodate material displaced from the neighbourhood of the foundation. This deformation of the soil is particularly important when the material is very compressible, or is confined as in the case of deep foundations, because the spread of the state of plastic equilibrium to the upper part of the theoretical failure zones is then usually prevented (local shear failure) and thus leads to a smaller bearing capacity than estimated. An analysis of this difficult problem has so far only been attempted on the basis of highly idealized analytical models; the foundation pressure is assumed to be normal to the face so that the major principal stresses and deformations of the material occur in the same direction (expansion under internal pressure) and the boundary (failure surface) between the plastic and elastic zones is parallel to the foundation face. The maximum pressure can then be found from the equilibrium at the failure surface, based on the strength and deformation properties of the material; the resulting pressure on the foundation shaft could be used to estimate the earth pressure coefficient K_s (p. 307).

In this way a rough estimate of the bearing capacity of a deep circular foundation (pile) in cohesive material with internal friction was obtained by Terzaghi (1925). He neglected elastic deformations and represented the foundation as a vertical cylinder and cone under internal pressure, with the stress conditions in the undeformed region being governed by the earth pressure coefficient at rest, K_0. For purely cohesive material of zero weight a solution of the same problem has also been obtained by Bishop, Hill and Mott (1945). They neglected consolidation deformations and represented the foundation base by the limits of a vertical cylinder or a sphere under internal pressure. A similar approach for a deep strip foundation is used in this article (p. 312) and is based on the simple analogue of a horizontal cylinder (representing the foundation base) under internal pressure.

Observations of the ground movements at failure have shown (Meyerhof, 1950) that the assumed deformations occur in practice usually only at some distance from the foundation,

and that in the neighbourhood of the foundation the mechanism of failure is similar to that estimated on the basis of a rigid material. It would therefore appear to be preferable, at present, to use the bearing capacity theory just derived for general shear failure ; and to take the compressibility of the material leading to local shear failure into account by an empirical reduction of the shearing strength using a compressibility factor (p. 320).
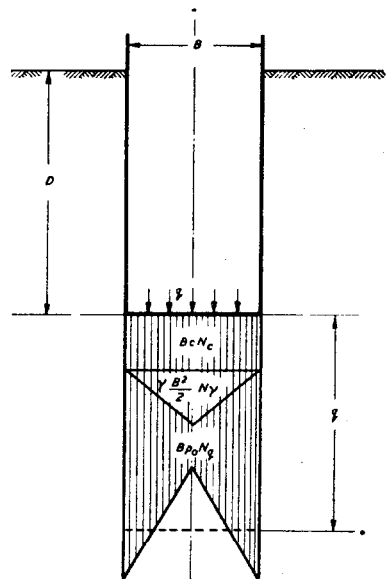
Distribution of contact pressure at failure

The theoretical distribution of the contact pressure on the base of a foundation at the ultimate bearing capacity, can be determined approximately from the corresponding distribution of the passive earth pressure on the central zone of material below the base (Terzaghi, 1943). In deriving the part of the bearing capacity. q', due to the cohesion and the equivalent free surface stresses it was assumed that the material in the plastic zones had no weight. The contact pressure due to the cohesion (N_c component) is therefore uniformly distributed, while the pressure due to the forces on the foundation shaft, and any adjacent soil wedges governing the equivalent free surface stresses (N_q component), is distributed in the same way as these stresses are distributed over the equivalent free surface. Since these stresses increase with depth from a minimum at the upper end of the composite failure surface to a maximum at the foundation edge, the corresponding contact pressure increases from the foundation centre to the edge. Similarly in estimating the part of the bearing capacity, q'', due to the weight of the material it was assumed that the cohesion and the equivalent free surface stresses were zero. The contact pressure due to the weight (N_γ component) increases therefore in direct proportion to the distance from the foundation edge.

The general results of this approach may be illustrated by the theoretical contact pressure for the base resistance of a rough strip foundation. The contact pressure distribution of a shallow foundation is found to be trapezoidal with a maximum at the foundation centre, and consists of a uniform N_c component, two relatively small triangular N_q components, and a triangular N_γ component (Fig. 6 (*a*)). With greater foundation depth the N_q component increases more rapidly than the N_c and N_γ components, which do not increase beyond a depth corresponding to $\beta = 90°$. The contact pressure distribution for a deep foundation is therefore inverted trapezoidal with a maximum at the foundation edge, and consists of a uniform N_c component, two relatively large N_q components, and a triangular N_γ component (Fig. 6 (*b*)). At very great foundation depths, when in practice the N_c and N_γ components can be neglected compared with the N_q component, the contact pressure is likely to be uniformly distributed because the height of the failure surface is then small compared with the depth of the foundation, so that the stress p_0 (governing the N_q component) is practically uniform.

Figs 6
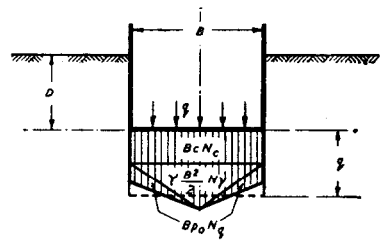
(a) SHALLOW FOUNDATION

(b) DEEP FOUNDATION

Distribution of contact pressure on base of rough strip foundation at failure

U

BEARING   CAPACITY   OF   PURELY
  COHESIVE   MATERIAL

Many materials have practically no internal friction ($\phi = 0$) and their shearing strength is closely represented by $s = c$ (from equation 1) with consequent simplification of the analysis as shown below.

Strip foundation

At the ultimate bearing capacity of a strip foundation with a rough base, the region above the composite failure surface can in general be divided into radial and plane shear zones on each side of a central elastic zone below the base (Figs 7); for a perfectly smooth base the latter zone has to be replaced by two plane shear zones as before. The bearing capacity can be represented by the equation

$$q = cN_c + p_0 \quad . \quad . \quad (20)$$

since for $\phi = 0$ the factor $N_q = 1$ and $N_\gamma = 0$ (equation 2). Further,

$$N_c = 3\,\pi/2 + 2\beta + 1 +$$
$$\sqrt{1 - m^2} - \cos^{-1}m \quad . \quad . \quad . \quad (21)$$

from equations (6) and (11) with symbols as before.

The factor N_c, which is independent of the degree of adhesion of the material on the base, is directly proportional to the inclination of the equivalent free surface. The factor has been calculated for the lower and upper limits ($m = 0$ and $m = 1$, respectively) of the shearing stress on the equivalent free surface within practical limits of β (Fig. 8). For $m = 0$ the factor varies from a minimum of $N_c = 2$ (compression test, $\beta = -90°$) to a maximum of $N_c = 3\pi + 2 = 11.42$ (completely embedded anchor beam, $\beta = 180°$); for $m = 1$ the factors are greater by $\pi/2 - 1 = 0.57$ than the corresponding lower factors.

To avoid estimating in every case the equivalent free surface stresses, and checking the resistance beyond the failure surface (p. 307), a complete solution has been obtained from the above factors for the bearing capacity of a strip foundation at depth D. It is given by the equation

$$q = cN_{cq} + K_s\gamma D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

from equation (5), where K_s can be taken as unity and N_{cq} is the resultant bearing capacity factor, which depends on N_c and N_q, the latter contributing relatively little.

The results of this analysis are given in Fig. 9 for a strip foundation with a rough base, and the limiting conditions of a perfectly smooth and a perfectly rough shaft ($c_a = 0$ and $c_a = c$, respectively). The corresponding factors increase with foundation depth at a decreasing rate from a minimum of $N_{cq} = \pi + 2 = 5.14$, for a surface foundation to a

Figs 7

Determination of N_c and N_{cr}

(a) SHALLOW FOUNDATION

Perfectly smooth | Perfectly rough
     shaft | shaft
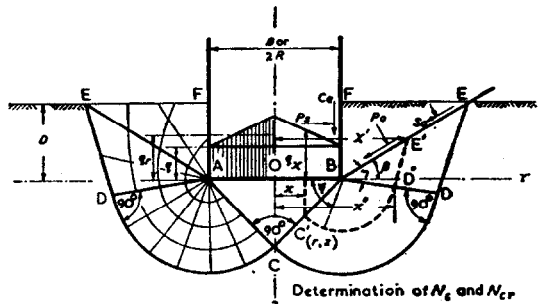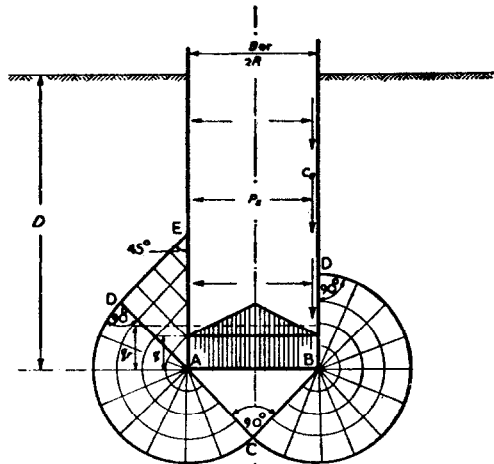
(b) DEEP FOUNDATION

Plastic zones near foundation with rough base in purely cohesive material

maximum of $N_{cq} = 2\pi + 2 = 8.28$ $(c_a = 0)$ and $5\pi/2 + 1 = 8.85$ $(c_a = c)$ for a deep foundation. It is of interest to note that in both cases the maximum resistance is theoretically reached at a depth of about twice the foundation width, and remains constant for greater depths. To the base or point resistance of a foundation with a rough shaft must be added the skin friction to obtain the total bearing capacity, for which the factor is shown in Fig. 9 for a perfectly rough shaft. The theoretical contact pressure distribution on the base at failure is uniform for all foundation depths (p. 311).

A rough estimate of the effect of the deformation characteristics of the material (leading to local shear failure) on the above results (based on general shear failure), is obtained by assuming that a deep foundation base can be represented by the analytical model of a horizontal cylinder under internal pressure (p. 310). A solution of this problem has been obtained by Bishop, Hill, and Mott (1945) who found that the maximum pressure is given by the equation

$$p = c \left(\log_e E_0/3c + 1\right) \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$
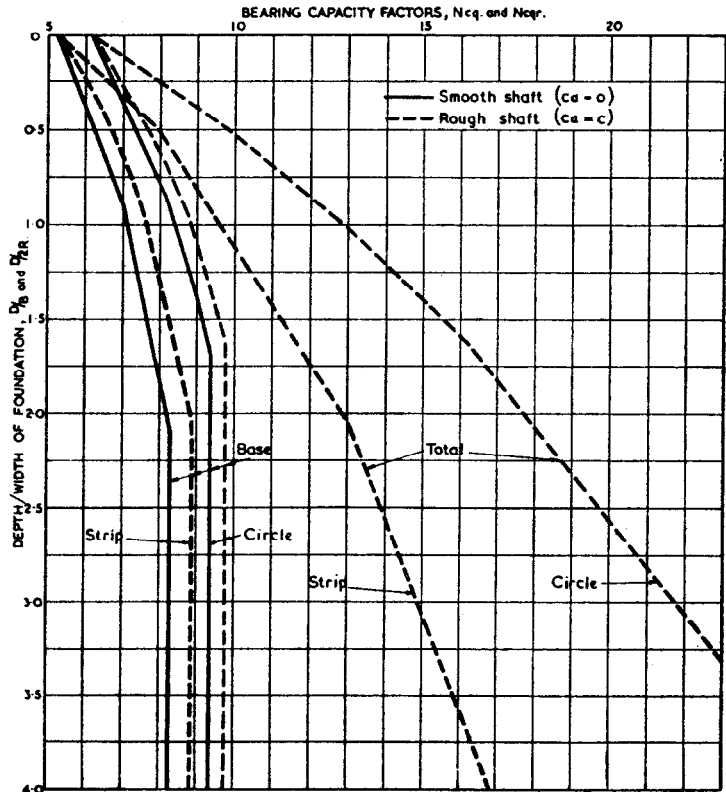
where E_0 denotes (initial tangent) modulus of elasticity.

Fig. 8
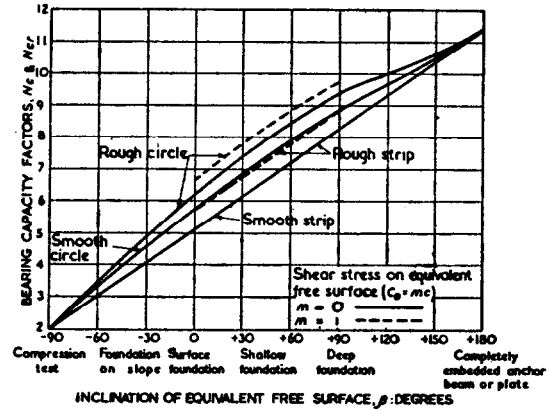
General bearing capacity factors for strip and circular foundations in purely cohesive material

Fig. 9

Bearing capacity factors for strip and circular foundations in purely cohesive material

This expression can be modified by adding the vertical component, c, of the circumferential shearing resistance on the cylinder and the overburden pressure γD to obtain the bearing capacity in a form similar to that of equation (22),

namely
$$q = cN_{cq} + \gamma D$$

where
$$N_{cq} = \log_e E_0/3c + 2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

For fully saturated clay E_0/c varies from about 100 to 500 in practice, giving N_{cq} equal to 5·50 to 7·00 compared with the present result of 8·28 ($c_a = 0$) for a rigid material. Since a similar approach when modified for a strip footing on the surface of a rigid material gives N_{cq} equal to 4·57, compared with Prandtl's value of 5·14, the results of the analytical model are somewhat too low. This approximate alternative method shows, however, that the compressibility of clay may reduce the bearing capacity of a deep strip foundation by up to approximately 20 per cent.

Circular foundation

At the ultimate bearing capacity of a circular foundation plastic flow of the material occurs in both horizontal and vertical (radial) planes. Normal to the radial planes act hoop stresses, which in accordance with Coulomb-Mohr's theory are equal to the minor principal stresses. On that basis it can be shown (Hencky, 1923) that in radial planes the plastic zones and composite failure surface are similar in shape to, and somewhat smaller in size than, those in transverse planes of a corresponding strip foundation. As a first approximation to the solution of the problem it will therefore be assumed that the two cases are identical, so that for a rough base the plastic region can in general be divided into annular radial and plane shear zones around a central elastic zone below the base (Figs 7); for a perfectly smooth base the latter zone has to be replaced by an annular plane shear zone as before.

The bearing capacity can be represented by
$$q_r = cN_{cr} + p_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where N_{cr} is the bearing capacity factor for a circular foundation.

The procedure of determining this factor is similar to that just outlined for a strip foundation and may be illustrated by estimating the bearing capacity of a circular foundation with a rough base of diameter $2R$ (Figs 7). The differential equations for the stresses in terms of cylindrical coordinates (r, z) were derived by Hencky (1923) and, so far as they relate to the present problem, they have been solved by the Author to give an expression for the contact pressure at failure q_x at any radius $r = x$, namely

$$q_x = q + c \left(\log_e \frac{x'}{x} - \int_{C'}^{E'} \frac{dz}{r} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$= q + \Delta q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26a)$$

where q denotes average contact pressure at failure (bearing capacity) of similar strip foundation (equation 20),

Δq denotes contact pressure due to hoop stresses at failure,

x and x' are radial coordinates of C' at beginning and E' at end, respectively, of the slip line $C'D'E'$ (parallel to the failure surface CDE) governing the contact pressure q_x.

Since
$$\int_{C'}^{E'} \frac{dz}{r} = a \int_{45°}^{180° + \beta - \eta} \frac{\cos \psi}{1 - a \cos \psi} \, d\psi - \cot (\eta - \beta) \log_e \frac{x'}{x''} \quad . \quad . \quad . \quad . \quad (27)$$

$$x' = R(1 + a \cos \beta/\cos \eta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

and
$$x'' = R\{1 + a \cos (\beta - \eta)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where
$$a = \sqrt{2}(1 - x/R),$$

ψ denotes angle at **B** between **AB** and point (r, z) (Figs 7) and other symbols as before, hence Δq, after substituting equations (27) to (29) into (26) and integrating, equals

$$c \left[\begin{array}{l} \log_e \dfrac{1 + a \cos \beta / \cos \eta}{1 - a/\sqrt{2}} + \cot (\eta - \beta) \log_e \dfrac{1 + a \cos \beta / \cos \eta}{1 + a \cos (\eta - \beta)} + \dfrac{3\pi}{4} + \beta - \eta \\[3mm] - \dfrac{2}{\sqrt{1 - a^2}} \left[\tan^{-1} \left\{ \sqrt{\dfrac{1 + a}{1 - a}} \cot \left(\dfrac{\eta - \beta}{2} \right) \right\} - \tan^{-1} \left\{ \sqrt{\dfrac{1 + a}{1 - a}} (\sqrt{2} - 1) \right\} \right] \end{array} \right] \quad (30)$$

for $a < 1$.

For $a > 1$ the last term of equation (30) is replaced by

$$- \frac{2}{\sqrt{a^2 - 1}} \left[\coth^{-1} \left\{ \sqrt{\frac{a + 1}{a - 1}} \cot \left(\frac{\eta - \beta}{2} \right) \right\} - \coth^{-1} \left\{ \sqrt{\frac{a + 1}{a - 1}} (\sqrt{2} - 1) \right\} \right].$$

Hence the bearing capacity is given by the equation

$$q_r = q + \frac{2}{R^2} \int_0^R \Delta q x \, dx \qquad \quad . \qquad . \quad . \quad . \quad . \quad . \quad (31)$$

from equation (26a) or

$$q_r = c N_{cr} + p_0$$

from equation (25) where $\quad N_{cr} = N_c + \dfrac{2}{cR^2} \int_0^R \Delta q x \, dx \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$

from equations (20) and (31), which integration must be carried out numerically with Δq given by equation (30).

The expression for Δq of a perfectly smooth circular foundation was found to be more cumbersome than that given above and will be omitted; its magnitude is about one-half of that for a perfectly rough foundation so that the factor N_{cr} is affected by the roughness of the base. The results of this analysis are shown in Fig. 8 for the limits of $m = 0$ and $m = 1$ as before. The bearing capacity factors are almost directly proportional to β and are generally greater than those of a strip foundation on account of the hoop stresses. At the minimum ($N_{cr} = 2$) and the maximum ($N_{cr} = 3\pi + 2 = 11 \cdot 42$) the corresponding factors are the same because the hoop stresses are then neutralized. For a perfectly smooth foundation on the surface the present value of $N_{cr} = 5 \cdot 71$ may be compared with the solution of $5 \cdot 68$ obtained by a numerical step-by-step computation (Ishlinsky, 1944). The approximate estimate on the basis of the present theory agrees therefore within $0 \cdot 50$ per cent in this case. If the present value of $N_{cr} = 6 \cdot 18$ for a perfectly rough foundation on the surface is subject to the same percentage error, this factor would become $6 \cdot 14 = \pi + 3$, exactly.

The bearing capacity of a circular foundation at depth D is determined from the above factors in the same way as outlined for a strip foundation, with the additional allowance for the effect of the hoop stresses. It is given by the expression

$$q_r = c N_{cqr} + K_s \gamma D \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where N_{cqr} is the resultant bearing capacity factor for a circular foundation and $K_s \simeq 1$ as before.

The results of this analysis are given in Fig. 9 for a circular foundation with a rough base and the limiting conditions of a perfectly smooth and a perfectly rough shaft ($c_a = 0$ and $c_a = c$, respectively). The corresponding factors increase with depth from a minimum of $N_{cqr} = 6 \cdot 18$ (probably $\pi + 3$), for a surface foundation to a maximum of $N_{cqr} = 9 \cdot 34$ ($ca = 0$) and $9 \cdot 74$ ($ca = c$) for a deep foundation beyond a depth of about twice the foundation diameter. To the base or point resistance of a foundation with a rough shaft must be added the skin friction to obtain the total bearing capacity, for which the factor is shown in Fig. 9 for a perfectly rough shaft. It is of interest to note that the factors for a circular

foundation have similar relations with depth to those found for a strip foundation and are about 20 per cent greater for a shallow foundation and about 10 per cent greater for a deep foundation.

The theoretical contact pressure distribution on the base at failure increases in accordance with equation (26) from a minimum value (equal to that of a strip foundation) at the perimeter to a maximum value at the centre of the foundation. This pressure distribution is practically trapezoidal for all foundation depths as indicated in Figs 7.

A rough estimate of the effect of the deformation characteristics can be obtained as before by assuming that a deep foundation base can be represented by the analytical model of a sphere under internal pressure (p. 310). A solution of this problem has been obtained by Bishop, Hill and Mott (1945) who found the expression for the maximum pressure to be

$$p = \frac{4}{3} c \left(\log_e \frac{E_0}{3c} + 1 \right), \qquad \ldots \ldots \ldots \quad (34)$$

which can be modified as before to obtain the bearing capacity in the form of equation (33), i.e.
$$q_r = cN_{cqr} + \gamma D$$
where
$$N_{cqr} = \frac{4}{3} \left(\log_e \frac{E_0}{3c} + 1{\cdot}75 \right) \qquad \ldots \ldots \ldots \quad (35)$$

with symbols as before.

For fully saturated clay with the previous limits of E_0/c the factor N_{cqr} varies from 7·00 to 9·00 compared with the present result of 9·34 ($c_a = 0$) for a rigid material. According to this approximate method the compressibility of clay may thus reduce the bearing capacity of a deep circular foundation by up to about 20 per cent as found for a deep strip foundation.

Rectangular and square foundations

On the assumption just made, that the plastic zones, and composite failure surface of strip and circular foundations are identical in cross section, a solution can be obtained for the bearing capacity of a rectangular foundation of length L and width B with semi-circular ends of radius $R = B/2$. Since the plastic zones are then continuous, the stresses in the central portion of length $L-B$ are the same as those of a strip foundation (p. 312) with the addition of longitudinal stresses which do not, however, affect the plastic equilibrium. These latter stresses are equal to the hoop stresses from the two end portions in which the stresses are identical to those of a circular foundation (p. 314).

The bearing capacity of a rectangular foundation at depth D is given by the equation
$$q_l = cN_{cql} + K_s\gamma D \qquad \ldots \ldots \ldots \ldots \quad (36)$$
where N_{cql} is the resultant bearing capacity factor for a rectangular foundation. Using equations (21) and (33) in the central and end portions of the foundation, respectively, it can be shown that

$$N_{cql} = \left[1 + \left(\frac{N_{cqr}}{N_{cq}} - 1 \right)\left(\frac{\pi}{4} + 0{\cdot}17 \frac{B}{L} \right)\frac{B}{L} \right]N_{cq} \qquad \ldots \ldots \quad (37)$$

$$= \left[1 + \left(\frac{N_{cqr}}{N_{cq}} - 1 \right)\frac{B}{L} \right] N_{cq}, \text{ very nearly,} \qquad \ldots \ldots \quad (37a)$$

$$\text{or } N_{cql} = \lambda N_{cq} \qquad \ldots \ldots \ldots \ldots \quad (37b)$$

where N_{cq} and N_{cqr} are the factors for a strip and circular foundation, respectively, (Fig. 9) and λ denotes the shape factor whose value is given within equations (37) and (37a).

A theoretical solution for the bearing capacity, q_s, of a square foundation is not yet available but it is not likely to differ appreciably from that of a circular foundation so that

$$q_s = q_r, \text{ approximately,}$$

or
$$N_{cqs} \simeq N_{cqr}. \qquad \ldots \ldots \ldots \ldots \quad (38)$$

Using the same method the bearing capacity of a rectangular foundation with square ends is then given by equation (37a). Since the ratio N_{cqr}/N_{cq} varies from 1·10 to 1·20 depending on the foundation depth and roughness of the shaft (p. 314),

$$N_{cql} = (1 + 0.15\ B/L)\ N_{cq} \qquad \ldots \ldots \ldots \ldots \quad (39)$$

from equation (37a) on the average for any depth.

BEARING CAPACITY OF COHESIONLESS MATERIAL

Many materials have practically no cohesion ($c = 0$) and their shearing strength is closely given by $s = p \tan \phi$ (from equation 1) with consequent simplification of the analysis, as follows.

Strip foundation

The plastic zones above the composite failure surface are approximately as shown in Figs 1 and 2. The bearing capacity of a foundation with a rough base is given by the expression

$$q = \gamma \frac{B}{2} N_\gamma + p_0 N_q \qquad \ldots \ldots \ldots \ldots \quad (40)$$

since for $c = 0$ the factor $N_c = 0$ (equation 2) ; the first term becomes $\gamma \dfrac{B}{4} N_\gamma$ for a foundation with a perfectly smooth base. The factors N_γ (equation 13) and N_q (equation 11 with $c = 0$) were obtained earlier (Figs 4 and 5).

To avoid estimating the equivalent free surface stresses (p. 307) in every case, a complete solution has been obtained for the bearing capacity of a strip foundation at depth D. It is given by the equation

$$q = \gamma \frac{B}{2} N_{\gamma q} \qquad \ldots \ldots \ldots \ldots \ldots \quad (41)$$

from equation (5) where $N_{\gamma q}$ is the resultant bearing capacity factor, which depends on N_γ and N_q, the former contributing more at shallow depth and the latter more at greater depth.

The results of this analysis for various values of ϕ are given in Figs 10 and 11 for a strip foundation with a rough base and the limiting conditions of a perfectly smooth and a perfectly rough shaft ($\delta = 0$ and $\delta = \phi$, respectively). The corresponding factors increase rapidly with foundation depth. At zero depth all curves are tangential to the straight line giving the bearing capacity-depth relation for no shearing strength of the overburden, which is found to be too conservative in practice even for very shallow foundations. At greater foundation depths the factor N_q governs the bearing capacity and the resultant factor $N_{\gamma q}$ is therefore directly proportional to depth and the earth pressure coefficient K_s whose minimum theoretical values (K_s = active earth pressure coefficient) are also indicated. These curves may also be used for a foundation with a smooth base by dividing both coordinates by 2. To the base or point resistance of a foundation with a rough shaft must be added the skin friction, to obtain the total bearing capacity.

The above results show that the earth pressure coefficient, K_s, on the shaft within the failure zone has an important influence on the theoretical bearing capacity of cohesionless material. This coefficient depends mainly on the density, strength, and deformation characteristics of the material, the stress-strain history of the ground (which may be represented by the coefficient of earth pressure at rest, K_0), and the method of installing and physical characteristics of the foundation. The value of K_s lies between the appropriate active (minimum) and passive (maximum) earth pressure coefficients, and can at present only be obtained from the results of field tests (p. 326).

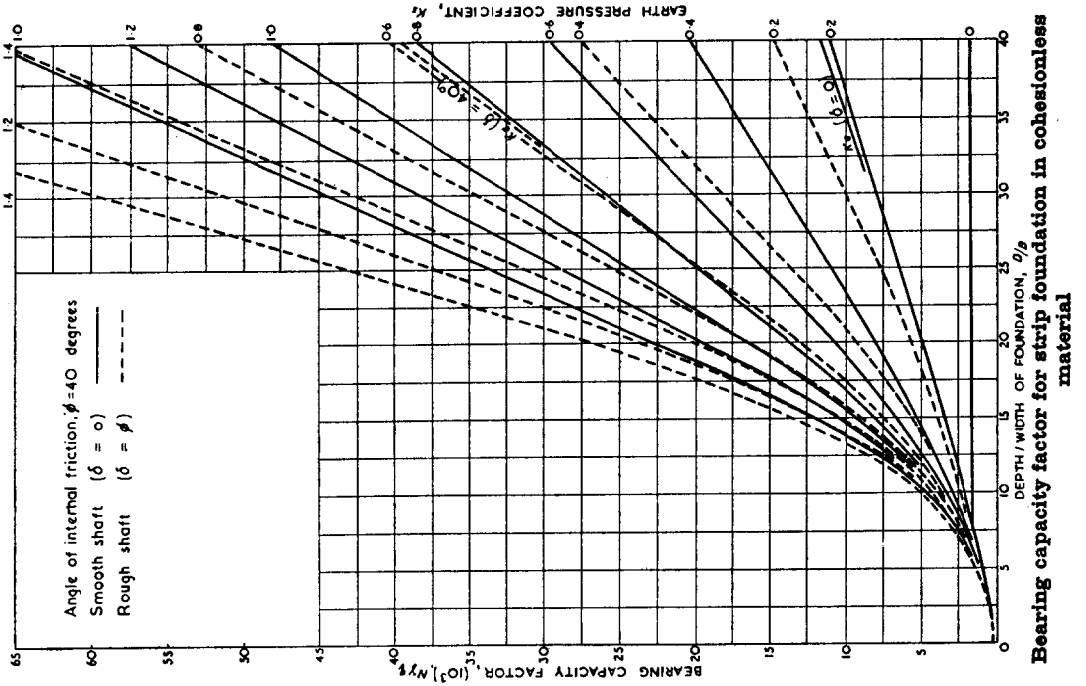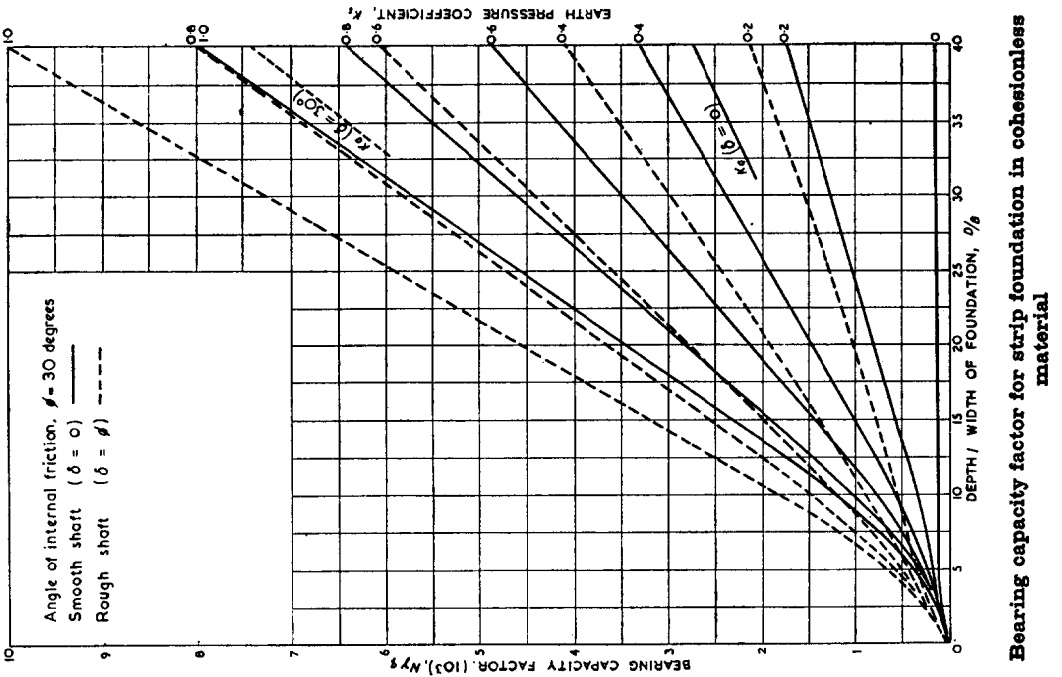The bearing capacity of cohesionless material is directly proportional to the (effective)

Fig. 11

Bearing capacity factor for strip foundation in cohesionless material

Fig. 10

Bearing capacity factor for strip foundation in cohesionless material

density γ (equation 41), which can vary from a maximum for a saturated material to a minimum of about one-half that value for a completely submerged material. In the latter case equation (41) becomes

$$q = \gamma' \frac{B}{2} N_{\gamma q} + \gamma_w D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where γ' denotes submerged density

and γ_w ,,   density of water.

The theoretical contact pressure distribution on the base at failure is trapezoidal, with a maximum at the centre for a shallow foundation and a maximum at the edge for a deep foundation (p. 311).

Circular foundation

Owing to mathematical difficulties only trial solutions have been made by the Author using a graphical method of radial sections on which the hoop stresses are equal to the minor principal stresses (p. 314). For a rough circular foundation on the surface, it was found that on radial planes the worst theoretical failure surface is approximately circular and cuts the ground level at a distance from the perimeter of about the footing diameter, which is the same as for purely cohesive material. The plastic zones are thus very much smaller than those of a strip foundation and do not seem to vary appreciably with the angle of internal friction.

In view of the approximately circular shape of the failure surface the friction circle method (Krey, 1936) has been used to estimate the bearing capacity factor $N_{\gamma r} = q_r/\gamma R$ for $\phi = 30°$ and $45°$. In both cases $N_{\gamma r}$ was found to be about one-half of the corresponding factor N_γ for a strip foundation. Similarly, for a circular foundation at great depth when $N_{\gamma r}$ can be neglected, it was found for the same values of ϕ that the factor $N_{qr} = q_r/K_s\gamma D$ was approximately twice that of the corresponding factor N_q for a strip foundation ; again the plastic zones were relatively small. The results of these computations are to some extent confirmed by test results (p. 326). On account of the tentative nature of the above approach it is, however, preferable at present to modify the theoretical bearing capacity factors of strip foundations in cohesionless material by an empirical shape factor λ for circular and other areas (p. 326).

PART II. INVESTIGATION OF BEARING CAPACITY IN PRACTICE

The theory of bearing capacity outlined in Part I had to be based on a number of simplifying assumption, relating mainly to the deformation characteristics of the material and the method of installing the foundation, the effect of which on the bearing capacity can at present only be taken into account on the basis of empirical evidence. The need for checking against experimental data is particularly important for materials with internal friction, owing to the major influence of the earth pressure coefficient on the shaft, and, in the case of circular foundations, the tentative theoretical approach. In order to provide this information the main results of laboratory and field loading tests on single foundations are analysed and compared with the theoretical estimates.

This investigation is restricted to the two main groups of materials, purely cohesive soil (clay), and cohesionless material (sand and gravel). In each case the principal types of foundations are considered, namely buried foundations and driven foundations. Buried foundations are installed by excavation of the soil and include shallow foundations (footings and rafts) and some types of deep foundations (piers and bored in-situ piles). Driven foundations are installed by displacement of the soil and include mainly piles (driven and in-situ piles with shell left in place). Foundations which are installed partly by excavation and

partly by displacement of the soil (caissons, some types of piers and in-situ piles) have a bearing capacity between that of similar buried and driven foundations and are outside the present scope.

BEARING CAPACITY OF PURELY COHESIVE MATERIAL

Buried foundations

The bearing capacity of model foundations in a number of undisturbed and remoulded clays of various strengths has been determined in the laboratory ; in many cases both base resistance and skin friction were measured simultaneously by a special device. Corrections were made for the effects of consolidation and rates of straining in relation to unconfined compression tests (Meigh, 1950) from which the shearing strength was determined.

The loading tests were carried out shortly after installing the foundation. For stiff clay general shear failure was usually observed with a definite rupture surface and a well defined bearing capacity, especially for shallow foundations, when the mechanism of failure was similar to that assumed in the theory. For soft clay, however, local shear failure occurred without noticeable rupture surface and the ultimate load was not well defined except for deep foundations. It is of interest to note that for deep foundations, particularly when they are rough, the skin friction is the major portion of the bearing capacity and is mobilized at about one-quarter to one-half of the penetration required for the maximum base resistance.

Comparison of the experimental base resistance of strip and circular foundations at various depths with the theoretical estimates (p. 312) indicates good agreement, especially for stiff clay (Fig. 12). For soft clay the great penetration required for mobilization of the shearing strength gives a greater bearing capacity than estimated for shallow foundations ($D/B < 1$), while local shear failure leads to a somewhat smaller bearing capacity for deep foundations. The maximum base resistance is reached at about twice the theoretical depth. Tests on rectangular areas of various length/width ratios and depths are also found to be in fair agreement with the theoretical relationship (Fig. 13). The bearing capacity factor for a square area is a little smaller than that for a circular area, and both are about 20 per cent greater than that for a long strip (see also Skempton, 1951).

The amount of adhesion on the shaft influenced the base resistance only to a small extent, as would be expected theoretically. The skin friction of a smooth (brass) shaft corresponded to an adhesion of about one-half of the shearing strength of the clay, which agreed with the results of direct shearing tests under the same conditions. In the case of a rough (sanded or concrete) shaft the full shearing strength was mobilized as would be expected.

The results of published field loading tests on foundations with zero or a small skin friction (shallow foundations and piers) have recently been summarized (Skempton, 1951), and are shown in Fig. 14, together with some additional data. This evidence, which is limited to soft clay, is in fair agreement with the theoretical estimates based on the average shearing strength in the theoretical failure zones. The bearing capacity of deep foundations is somewhat less than estimated as found in the laboratory tests on soft clay. In addition, some plate loading tests have been carried out at the bottom of boreholes in firm sandy clay (Ostenfeld, 1942) and stiff boulder clay (Mortensen, 1948) ; analysis of these results indicates that the bearing capacity was 9 to 10 times the direct shearing strength, which agrees reasonably well with the theory.

As the depth of foundations increases, the skin friction becomes of greater importance. Little published information exists on the magnitude of the skin friction of buried foundations in relation to the shearing strength of the soil. Some information about this problem has been obtained from an extensive series of field loading tests on bored piles in firm to stiff clay in the London area, carried out as part of the foundation research programme of the Building Research Station. The piles were formed by pouring, and in the case of long piles by ramming, fairly dry concrete into unlined auger boreholes. Details of the soil properties on two
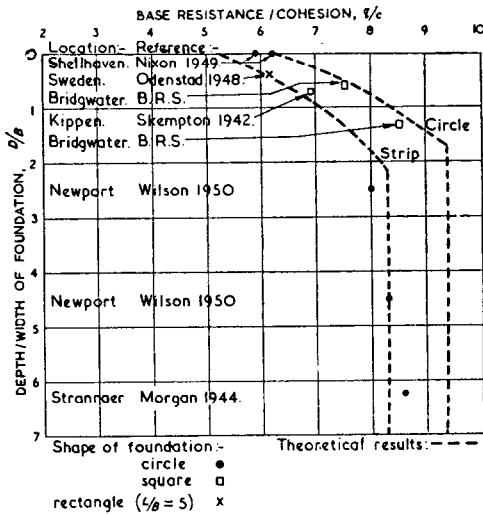
Fig. 12
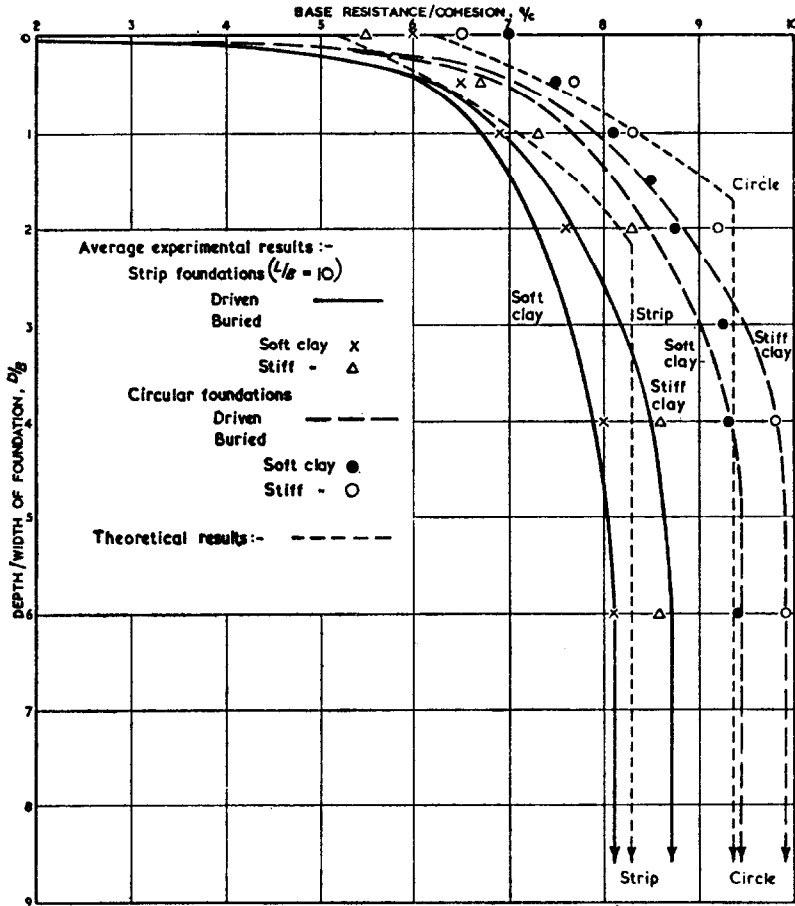
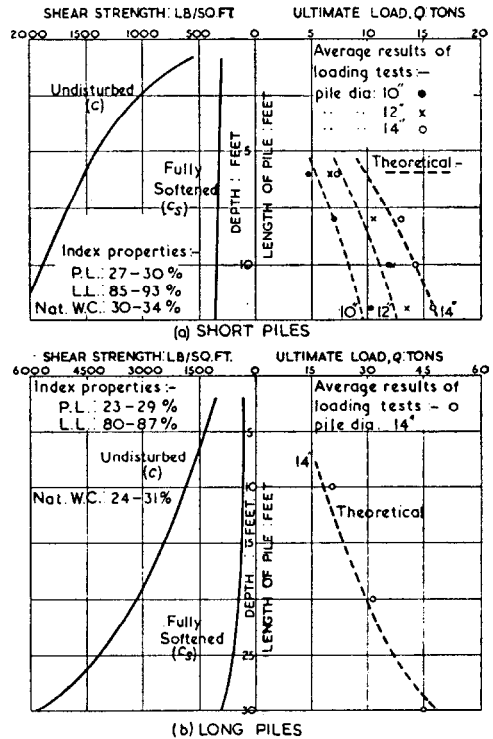Bearing capacity of buried and driven model foundations in clay

Fig. 13
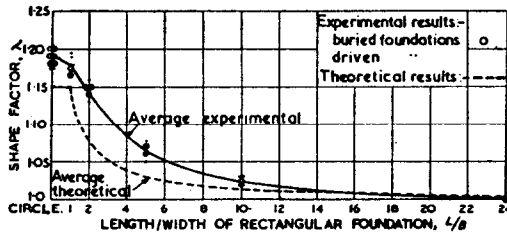
Shape factor of foundations in clay

Fig. 14   Figs 15

Results of field loading tests on foundations
in soft clay

Details of field loading tests on bored in-situ
piles in London clay

typical test sites are given in Figs 15, which also shows the average results of loading tests
carried out at about a month after casting the piles. Some of the piles were reloaded at an
interval of up to one year after the first tests without showing any significant increase in
the bearing capacity. Examination of the soil surrounding typical piles indicated that
water from the concrete had penetrated into the clay and softened it considerably. The
water content of the clay within about 2 inches of the pile surface increased rapidly towards
the shaft ; in a very thin layer adjacent to the pile the water content was by one tenth to one
fifth greater than the original amount and corresponded to that of softened clay under zero
overburden. The skin friction of the piles has therefore been estimated from the shearing
strength of the fully softened material, and has been added to the theoretical point resistance
based on the natural shearing strength to obtain the total estimated bearing capacity. In
view of the softening of the clay the theoretical skin friction was only of the same order of
magnitude as the theoretical point resistance, which is not affected by the local softening at
the base. Comparison of these estimates with the observed bearing capacities shows fair
agreement (Figs 15) and indirectly supports the theoretical bearing capacity factors.

It may be concluded from the laboratory and field loading tests that the theory enables
a reasonable estimate to be made of the bearing capacity of buried foundations in clay.
For deep foundations ($D/B > 1$) in soft clay the actual base resistance is somewhat less than
estimated, on account of the compressibility of the material leading to local shear failure.
In that case Terzaghi (1943) suggested the use of a reduced cohesion, $c' = \frac{2}{3} c$, in the estimates.
This approach may be generalized by introducing an empirical compressibility factor, κ,
such that $c' = \kappa c$. The above analysis of the test results on the basis of the present theory

indicates that for stiff clay $\kappa = 1$, and that for soft clay κ varies from about 1·00 for shallow foundations to about 0·90 for deep foundations. Unless the sides of the excavation are protected, the skin friction corresponds to that of the fully softened material and is therefore negligible for shallow foundations.

Driven foundations

The bearing capacity of driven (actually pushed) model foundations has been determined in the laboratory for clays of various strengths and sensitivities as before. Load-settlement curves of the tests were similar to those of buried foundations, but the bearing capacity was generally reached at less than one-half of the above settlement. This difference may be explained by the relationship between virgin deformation of the material (buried foundations), and recompression (driven foundations) when the bearing capacity up to the depth to which the foundation is driven has already been exceeded. Since driven foundations are always deep in practice (piles) and the settlement at the ultimate load is relatively small, the bearing capacity at any depth immediately after installation of the foundation can be obtained from a continuous penetration test, which forms the envelope of the individual loading test results.

Load-penetration curves of strip and circular foundations are given in Fig. 12. The experimental base resistance is in fair agreement with the theoretical estimates, which are approached with increasing strength of the clay and form the envelope of the experimental curves. For soft clay local shear failure gives a smaller bearing capacity as before. The effect of the shape of the foundation on the base resistance is also found to be in fair agreement with the estimates as shown by tests on rectangular areas of various length/width ratios (Fig. 13).

While the amount of adhesion influenced the base resistance only to a small extent as before, the skin friction varied between wide limits, depending mainly on the sensitivity of the soil and the time interval between driving and loading the foundation. For undisturbed insensitive clay, the skin friction immediately after installing the foundation was of the same order as that for similar buried foundations. Tests at some time after driving showed that the skin friction had increased somewhat due to consolidation of the soil, especially in the case of a concrete foundation (Fig. 16). In all cases the point resistance was practically unchanged.

For undisturbed sensitive clay, the skin friction immediately after installing the foundation corresponded to about one-half of the natural shearing strength, or to the fully remoulded strength, whichever was the greater. Field conditions would probably lead to complete remoulding of the material in all cases. The skin friction increased considerably with time due to age-hardening and consolidation of the material (Fig. 16) ; for a concrete foundation the consolidation was greater than for a foundation with an impermeable rough shaft. While the increase in skin friction due to age-hardening can be estimated from the results of corresponding compression tests, the probable consolidation cannot be estimated from soil tests at present, and in some experiments increased the skin friction beyond that corresponding to the undisturbed strength. The age-hardening effect was destroyed on further penetration of the foundation when only consolidation effects remained (Fig. 16). For remoulded clay the skin friction immediately after driving was the same as for buried foundations, and increased with time due to age-hardening and any consolidation, as for sensitive clay.

In all laboratory tests the skin friction of pulling out the foundation was generally found to be somewhat greater than that measured during pushing immediately before. The point resistance was hardly affected by the sensitivity of the clay and corresponded to the undisturbed strength, so that it can readily be estimated in all cases. As just indicated the more important skin friction depends, however, on so many factors that it can only be estimated

G. G. MEYERHOF

Fig. 16
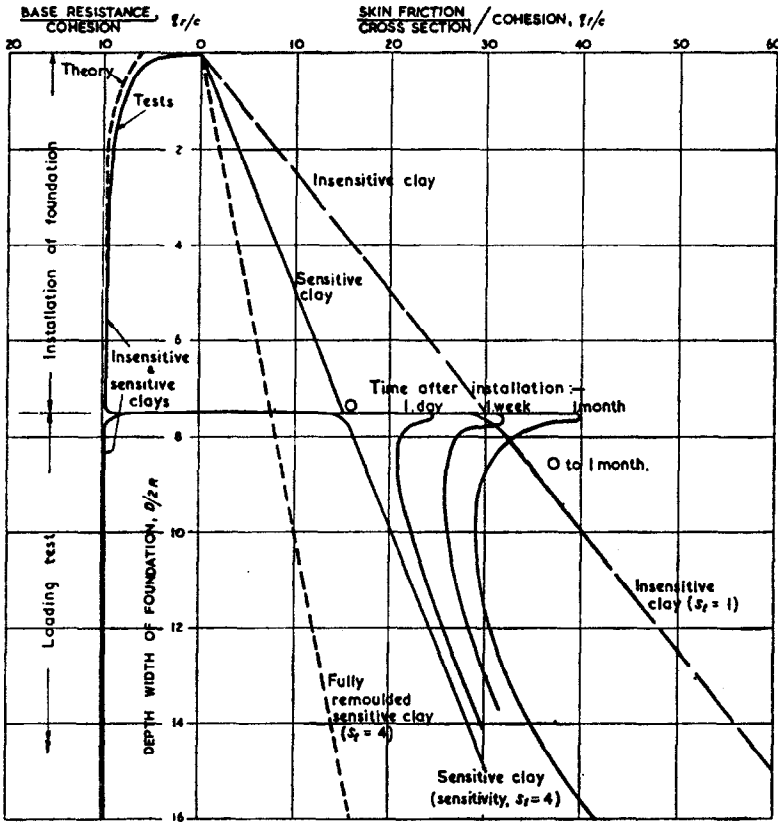
Bearing capacity of rough model piles driven in clay

very approximately from soil tests; the model tests on driven foundations must therefore be regarded as providing mainly qualitative information. Field loading tests on foundations driven into fairly uniform clay will therefore be analysed later in this article where sufficient data are available.
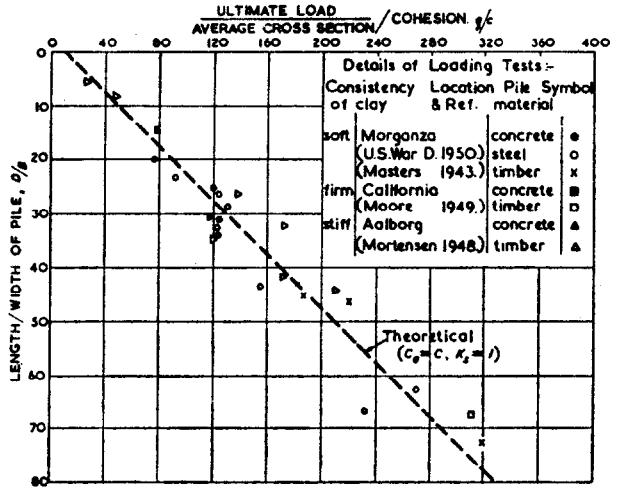
The results of published field loading tests on piles in insensitive clay are shown in Fig. 17, and are found to be in good agreement with the theoretical estimates based on the average shearing strength of the material, a perfectly rough shaft $(c_s = c)$ and an earth pressure coefficient $K_s = 1$. The bearing capacity of smooth (steel and timber) piles is similar to that of rough (concrete) piles, contrary to the difference found in the model tests because large piles are not as smooth. Where pulling tests have been made the skin friction was sometimes rather less than that estimated from soil tests with a correspondingly greater deduced point resistance. On the other hand, the mean point resistance obtained from the loading test results is approximately 10 times the shearing strength of the material, as shown by the intercept of the graph for zero foundation depth (Fig. 17), which agrees reasonably well with the theory.

Only a few results of field tests in sensitive clay are available and are shown in Fig. 18.

They indicate that immediately after driving of the piles the skin friction, and thus for practical purposes the bearing capacity, correspond to the shearing strength of the fully remoulded material. After a few months, however, the skin friction approaches and sometimes exceeds that corresponding to the undisturbed strength of the clay on account of age-hardening and consolidation of the material, the two contributions being similar in magnitude.

It may be concluded from the laboratory and field loading tests that the theory enables a reasonable estimate to be made of the bearing capacity of driven foundations in insensitive clay. The base resistance can be determined as for buried foundations and the skin friction corresponds to that of a rough foundation; the

Fig. 17

Results of field loading tests on driven piles in insensitive clay

shearing strength of the material is usually fully mobilized along the shaft within the present upper limit of an adhesion of about 1 ton per square foot observed in the field (Mortensen, 1948). For sensitive clay the base resistance can be determined as above; an estimate of the skin friction immediately after driving can be obtained from the remoulded shearing strength. The bearing capacity a few months after installing the foundation can usually be estimated from the undisturbed shearing strength of the material; until further data is published, it is in any given case advisable to carry out field loading tests as a check.
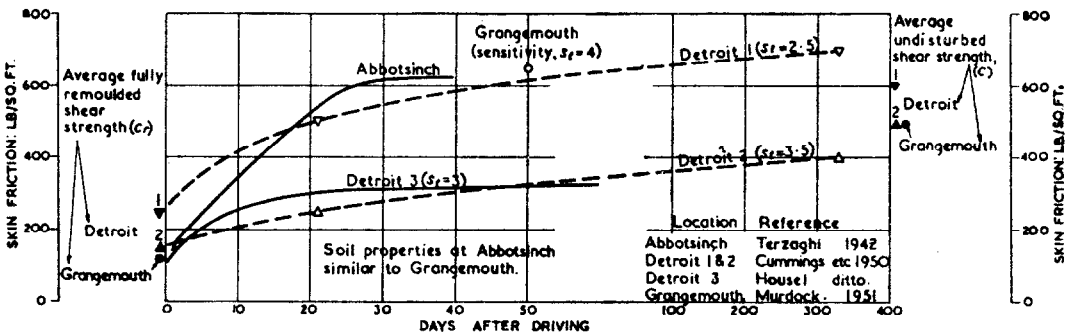
BEARING CAPACITY OF COHESIONLESS MATERIAL

Buried foundations

The bearing capacity of model foundations in dry and wet sands of various densities has been determined in the laboratory; as full details of this work are given elsewhere (Meyerhof, 1950), only the main results will be summarized here.

Fig. 18

Results of field loading tests on driven piles in sensitive clay

For dense sand general shear failure was usually observed with definite rupture surface and a well-defined bearing capacity. The mechanism of failure was similar to that assumed in the theory, and the extent of the rupture surface was much greater for strip than circular foundations ; at greater depths failure was more localised. For loose sand local shear failure occurred without noticeable rupture surface. The bearing capacity of deep foundations (even with a perfectly rough shaft) is mainly due to base resistance ; the skin friction is small and mobilized at about one-quarter to one-half of the penetration required for the maximum base resistance.

In order to compare the experimental base resistance with the theoretical estimates (p. 317), it is necessary to know the angle of internal friction of the material, the angle of skin friction, and the earth pressure coefficient on the shaft. The angles of internal and skin friction, which have been determined from direct shearing tests, vary with normal pressure on the shear plane and it has been shown (Meyerhof, 1948) that for shallow foundations the average normal pressure on the failure surface is of the order of one tenth of the bearing pressure, increasing to about one fifth of that pressure for deep foundations. The average earth pressure coefficient on the shaft within the failure zone can at present only be deduced from the observed skin friction (using equation 16) and was found to vary from about 1·00 for dense sand to about 0·50 for loose sand. It is of interest to note that this coefficient was of the order of one-half of the earth pressure coefficient at rest (determined by ancillary torsion tests on buried piles) in dense sand, and approximately equal to that coefficient in loose sand.

The theoretical base resistance of strip foundations at various depths has been estimated from the above data, and is shown compared with the experimental results in Fig. 19. While good agreement is obtained for shallow foundations, the actual bearing capacity is considerably less than estimated for deep foundations ($D/B > 5$), especially in loose sand, owing to the compressibility of the material leading to local shear failure. In that case Terzaghi (1943) suggested the use of a reduced internal friction tan, $\phi' = \frac{2}{3} \tan \phi$, in the estimates. By introducing an empirical compressibility factor κ, as before, such that tan $\phi' = \kappa \tan \phi$, the above analysis of the test results on the basis of the present theory indicates that κ varies from about 1·00 for shallow foundations to about 0·85 for deep foundations.

Tests on rectangular areas of constant width but various length/width ratios, and on similar circular areas, showed that for dense sand the bearing capacity decreases with smaller length/width ratios to a minimum for a circular area on the surface and at very shallow depth, and that the bearing capacity increases with smaller ratios to a maximum for a circular area at greater depths (Fig. 19). As the density and angle of internal friction of the sand decrease, the shape effect becomes less pronounced, and for loose sand the bearing capacity of foundations of constant width was found to be independent of the shape on the surface and at all depths. In view of the tentative nature of the theoretical analysis of circular foundations in cohesionless material (p. 319), it is advisable at present to modify the theoretical bearing capacity factors of strip foundations by an empirical shape factor λ, where λ is the ratio of the observed base resistance of a particular area to that of a long strip. This factor is shown in Fig. 20 for various values of ϕ. The bearing capacity of buried foundations in submerged sand and in sand with upward seepage, was found to be directly proportional to the effective density of the material, as would be expected theoretically.

While the bearing capacity of a perfectly smooth base was found to be one-half of that of a perfectly rough one in accordance with the theory, the amount of friction on the shaft influenced the base resistance only to a small extent as would be expected. The skin friction of a smooth (brass) shaft corresponded to one-half of the shearing strength of the sand, which agreed with the results of direct shearing tests under the same conditions ; in the case of a rough shaft the full shearing strength is likely to be mobilized.

Although a number of field loading tests on surface foundations and a few tests on buried foundations have been published, the only results where sufficient data for an analysis are

Fig. 19
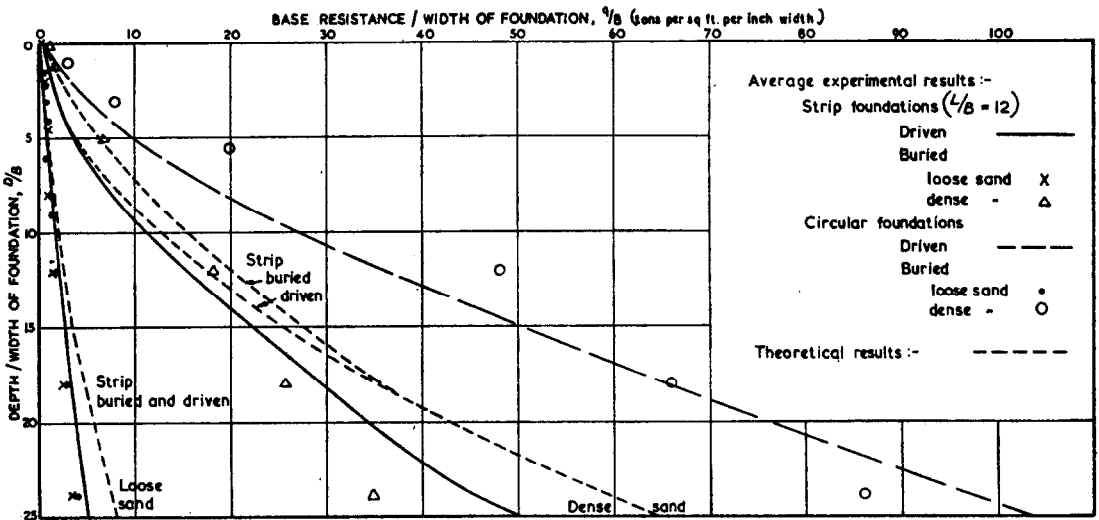
Bearing capacity of buried and driven model foundations in sand

available appear to be those shown in Fig. 21. This evidence, which is limited to surface foundations, is in fair agreement with the theoretical estimates based on the average shearing strength in the theoretical failure zones. When analysing loading tests below the surface a knowledge of the earth pressure coefficient K_s is also required, which can approximately be deduced from the skin friction measured in a penetration test (p. 329) ; this gives only a rough estimate of K_s, because the method of installing a buried foundation differs from that of a penetrometer

It may be concluded from the laboratory and very limited field loading tests available that the theory enables a reasonable estimate to be made of the bearing capacity of shallow foundations in cohesionless soil, provided the shearing strength and earth pressure coefficient K_s are known. The earth pressure coefficient can be deduced from the observed skin friction of model foundations in the laboratory and approximately from penetration tests in the field. For deep foundations $(D/B > 5)$, the actual base resistance is considerably less than estimated, on account of the effect of compressibility of the material leading to local shear failure ; in that case the shearing strength is reduced by an empirical compressibility factor of about 0·85. Even where the sides of the foundation are rough, the skin friction is small in proportion to the base resistance.

Driven foundations

The bearing capacity of driven (actually pushed) model foundations has been determined in the laboratory for dry and wet sands as before. Load-settlement curves of the tests were similar to those of buried foundations, and in view of the small settlement at the ultimate load, a continuous penetration test gives the bearing capacity of driven foundations at any depth. Load-penetration curves of strip and circular foundations are given in Fig. 19. The theoretical base resistance of strip foundations has been determined, as just outlined for buried foundations, and comparison with the experimental results shows good agreement at shallow depths. For deep foundations $(D/B > 5)$ the actual bearing capacity is less than estimated but the difference is smaller than for buried foundations, especially for loose sand,

W

Fig. 20
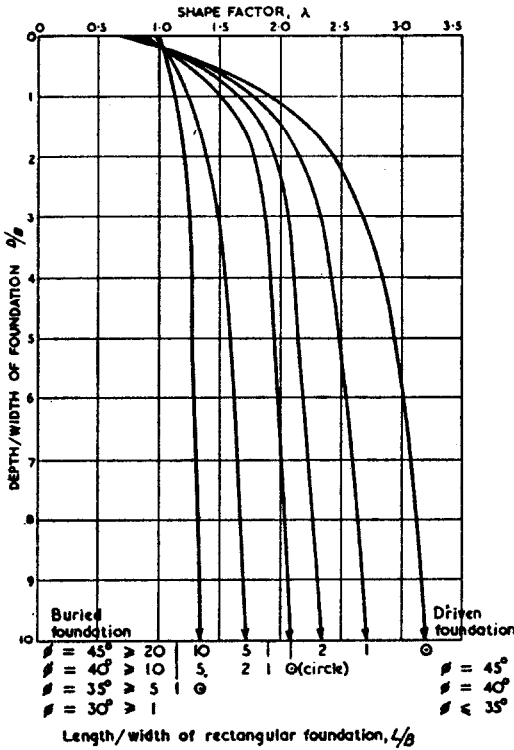
Shape factor of foundations in sand

because the influence of compressibility of the material is partly offset by the increased density after installing the foundation ; the corresponding empirical compressibility factor κ is therefore only 0·95.

Tests on rectangular areas of constant width but various length/width ratios, and similar circular areas, showed that the shape effect was similar to that found for buried foundations as shown by the corresponding empirical shape factors (Fig. 20). The bearing capacity of driven foundations in submerged sand was found to be directly proportional to the effective density of the material, as would be expected theoretically.

The skin friction during driving the foundation was greater than the corresponding value for buried foundations, especially for loose sand, due to compression of the material during installation ot the foundation with a consequent increase of the earth pressure coefficient K_s ; the skin friction during pulling was less than that during driving immediately before. While the skin friction of a smooth (brass) shaft corresponded to one-half of the shearing strength of the sand, in accordance with the results of direct shearing tests as before, the full shearing strength is likely to be mobilized in the case of a rough shaft.
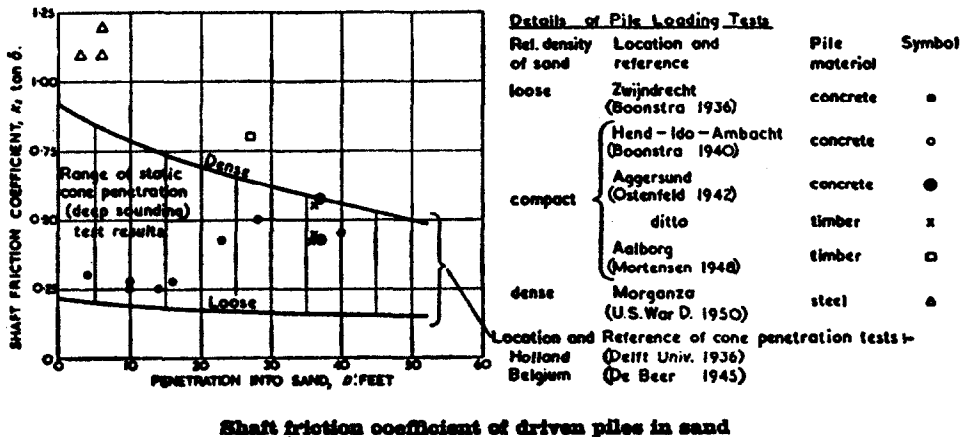
Fig. 21
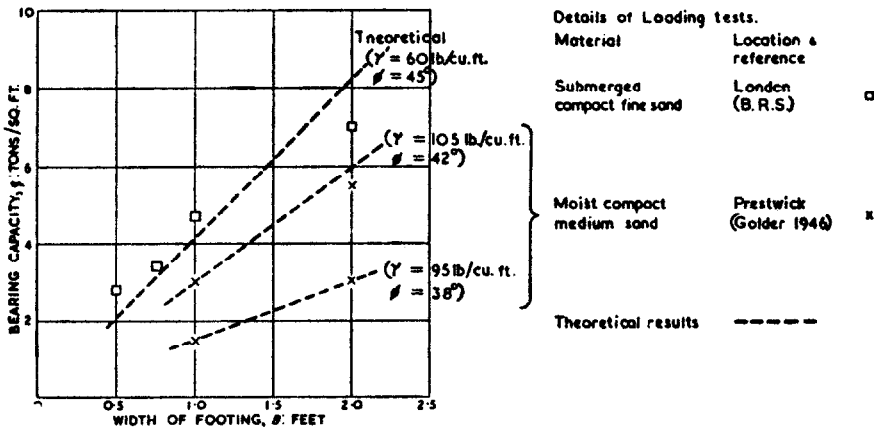
Results of field loading tests on footings in sand

No field loading tests on driven foundations appear to have been published with the results of shearing tests on the same material, owing to the difficulty of obtaining undisturbed samples of cohesionless soil. For foundations below the surface it is also necessary to know the earth pressure coefficient K_s, which can be deduced from the observed skin friction of piles (using equation 16) as was done above for the model tests in the laboratory. To obtain some information about the magnitude of this coefficient in the field, the published skin friction measurements on piles have been analysed ; the results are given in Fig. 22 in relation to the depth of penetration into cohesionless material. Since the angle of skin friction, tan δ, lies usually between 0·50 (loose sand) and 1·00 (dense sand), the coefficient K_s is seen to vary from about 0·50 for loose sand to about 1·00 for dense material. Fig. 22 also shows the results of a similar analysis of the skin friction observe in static cone penetration (deep sounding) tests and indicates that the coefficient K_s is somewhat smaller than that deduced from pile tests. This result is to be expected, because the lateral compression of the material during a penetration test is considerably less than that during installation of a full-sized pile, so that a penetration test furnishes results on the safe side.

This approach has recently been used to estimate the bearing capacity of piles driven through a thin bed of stiff clay, into compact to dense uniform fine sand, in London. On the basis of the shearing strength obtained from triaxial tests on slightly disturbed samples of the sand, and the earth pressure coefficient deduced from penetration tests, the theoretical bearing capacity of the piles was estimated to be about 150 tons per square foot when driven into the upper compact portion of the sand, and about 220 tons per square foot when driven into the top of the lower dense portion. Loading tests on typical piles gave 160 and 240 tons per square foot for the upper and lower portions, respectively, which is in good agreement with the estimates.

In view of the difficulty and expense of obtaining undisturbed samples of cohesionless material, it is frequently preferable to use mainly the results of cone penetration tests for an estimate of the bearing capacity of piles, by extrapolating the results either directly or by means of the proposed theory. In the first method it is assumed that the (minimum) unit cone resistance near base level and the average unit skin friction along the shaft are the same for the small-scale (penetrometer) and full-scale piles with similar point and shaft ; thus in the above mentioned loading tests extrapolation of the penetration test results gave an estimated bearing capacity of 200 and 300 tons per square foot for piles at the upper and lower levels, respectively. Extensive penetration and pile loading tests (Boonstra, 1936 and 1940,

Fig. 22

Details of Pile Loading Tests			
Rel. density of sand	Location and reference	Pile material	Symbol
loose	Zwijndrecht (Boonstra 1936)	concrete	●
compact	Hend – Ido – Ambacht (Boonstra 1940)	concrete	○
	Aggersund (Ostenfeld 1942)	concrete	⬤
	ditto	timber	✕
	Aalborg (Mortensen 1948)	timber	▢
dense	Morganza (U.S. War D. 1950)	steel	▲

Location and Reference of cone penetration tests :-
Holland (Delft Univ. 1936)
Belgium (De Beer 1945)

Shaft friction coefficient of driven piles in sand

Plantema, 1948, Huizinga, 1951) indicate that this method is usually satisfactory, especially for a large penetration into a cohesionless stratum, when the bearing capacity is almost entirely due to the overburden effect (N_q factor).

For small penetrations ($D/2R < 10$) of piles into cohesionless material the above method is, however, frequently on the unsafe side because a penetrometer of diameter $2r$ considerably overestimates the embedment ratio $D/2r$ in comparison with that of $D/2R$ of a pile of diameter $2R$. In that case an extrapolation of the penetration test results by means of the theory of bearing capacity appears to be preferred; although it is more cumbersome than the first method, it can be applied to any penetration of the pile. Both the reduced angle of internal friction, ϕ', and the earth pressure coefficient, K_s, are deduced by means of the theory from the measured cone resistance and skin friction (embedment ratio $D/2r$) using Figs 10 and 11 in conjunction with Fig. 20 and equation (16). The bearing capacity of the pile is then estimated by the theory using these values of ϕ' and K_s, and the appropriate ratio $D/2R$. Application of this method to published field data showed good agreement; for large penetrations of the pile both methods give similar results as would be expected.

It may be concluded from the laboratory and field loading tests that the theory enables a reasonable estimate to be made of the bearing capacity of driven foundations in cohesionless soil. While it is best to determine the shearing strength of the material from undisturbed samples, it is usually more convenient to assess that strength from an analysis of the results of penetration tests, which are required in any case to estimate the earth pressure coefficient K_s from the observed skin friction. This procedure is particularly useful for foundations with a small penetration into cohesionless material. For foundations with a large penetration it is more convenient to extrapolate the results of cone penetration tests directly. Even where the sides of the foundation are perfectly rough, the skin friction is small in proportion to the base resistance.

CONCLUSION

A theory of bearing capacity has been developed, on the basis of plasticity, by extending the previous analysis for a surface footing to shallow and deep foundations in a cohesive material with internal friction. The theoretical results can be represented by bearing capacity factors in terms of the mechanical properties of the material and the physical characteristics of the foundation. The analysis indicates that in general the bearing capacity increases with size, depth, and roughness of the base, and depends on the shape of the foundation. The bearing capacity is reduced by compressibility of the material leading to local shear failure, and this effect is at present best taken into account by an empirical reduction of the shearing strength. The influence on the soil properties of the method of installing the foundation is also based on empirical evidence.

Analysis of the main results of laboratory and field loading tests on buried and driven foundations in clay and sand shows reasonable agreement with the proposed theory in the case of shallow foundations. For deep foundations the actual base resistance is less than estimated, on account of local shear failure, and an empirical compressibility factor is introduced by which the shearing strength is reduced. In view of the difficulty of procuring undisturbed samples of cohesionless soil, and the necessity of obtaining the earth pressure coefficient on the shaft from field observations, it is at present best to obtain this information from an analysis of the results of cone penetration tests.

ACKNOWLEDGEMENT

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